

DR. BABASAHEB AMBEDKAR TECHNOLOGICAL UNIVERSITY, LONERE

Bachelor of Technology (Civil Engineering) SEMESTER - 3 Summer 2025 (Supply.)

Course :Bachelor of Technology (Civil Engineering) Branch : Engineering and Technology

Semester : SEMESTER - 3

Subject Code & Name: BTBS301 - ENGINEERING MATHEMATICS - III

Time : 3 Hours]

[Total Marks : 60

Instructions to the Students:

1. Each question carries 12 marks.
2. Question No. 1 will be compulsory and include objective-type questions.
3. Candidates are required to attempt any four questions from Question No. 2 to Question No.6
4. Use of non-programmable scientific calculators is allowed.
5. Assume suitable data wherever necessary and mention it clearly.

Q1. Objective type questions. (Compulsory Question)

12

- 1 The Laplace transform of $f(t) = e^{2t}t$ is
(a) $\frac{1}{(s-2)^2}$ (b) $\frac{1}{(s^2-4)}$
(c) $\frac{1}{(s+2)^2}$ (d) $\frac{s}{(s-2)^2}$
- 2 $L\{\delta(t-a)\} =$
(a) e^{as} (b) e^{-as}
(c) e^s (d) e^{-s}
- 3 $L\left\{\int_0^t \int_0^t \int_0^t \cos au \, du \, du \, du\right\} =$
(a) $\frac{1}{s^2 + a^2}$ (b) $\frac{1}{s^2(s^2 + a^2)}$
(c) $\frac{a}{s^2 + a^2}$ (d) None of these
- 4 If $L^{-1}\{F(s)\} = f(t)$, then $L^{-1}\{e^{-as}F(s)\}$ is equal to
(a) $f(t-a)H(t-a)$ (b) $h(t)f(t-a)$
(c) $f(t)h(t-a)$ (d) None of these
- 5 $L^{-1}\left\{\frac{2}{s^2 - 6s + 5}\right\} =$
(a) $e^{3t} \sin 2t$ (b) $e^{3t} \cosh 2t$
(c) $e^{3t} \cos 2t$ (d) $e^{3t} \sinh 2t$
- 6 The fourier cosine transform of the function $f(t)$ is
(a) $F_c(s) = \int_0^\infty f(t) \cos st \, dt$ (b) $F_c(s) = \int_0^\infty f(t) \cos t \, dt$
(c) $F_c(s) = \int_0^\infty f(st) \cos t \, dt$ (d) None of these

7 If $F(s)$ is Fourier transform of $f(x)$ then Fourier transform of $f(ax)$ is

- (a) $\frac{1}{a}F\left(\frac{s}{a}\right)$ (b) $\frac{1}{a}F(s)$
 (c) $F\left(\frac{s}{a}\right)$ (d) None of these

8 The solution of $p + q = z$ is

- (a) $f(x + y, y + \log z) = 0$ (b) $f(x^2 + y^2 + z^2, xyz) = 0$
 (c) $f(xy, y \log z) = 0$ (d) $f(x - y, y - \log z) = 0$

9 Partial differential equation for one dimensional wave equation is

- (a) $U_{tt} = U_x$ (b) $U_{tt} = c^2 U_{xx}$
 (c) $U_x = c^2 U_{tt}$ (d) $U_{tt} = c^2 U_{xx}$

10 Which of the following is an analytic function

- (a) $f(z) = \sin z$ (b) $f(z) = \bar{z}$
 (c) $f(z) = \operatorname{Im}(z)$ (d) None

11 The value of integral $\int_c \frac{dz}{z - a} =$

- (a) $2\pi i$ (b) 2π
 (c) πi (d) $-2\pi i$

12 The value of integral $\int_c \frac{5z^2 - z + 3}{z - 1} dz$ where c is $|z| = \frac{1}{2}$

- (a) $2\pi i$ (b) 0
 (c) πi (d) $-2\pi i$

Q2. Solve the following.

A) Find $L\left\{e^{-2t} \int_0^t t \cos ht \, dt\right\}$

6

B) Find Laplace transform of $f(t) = te^{2t} \cos^2 t$

6

Q3. Solve the following.

A) Find $L^{-1}\left(\frac{s}{(s^2 + 1)(s^2 + 4)}\right)$

6

B) Use Laplace transform to solve

6

$\frac{dy}{dt} + 3y(t) + 2 \int_0^t y(t) dt = t$ with $y(0) = 0$

Q4. Solve Any Two of the following.

A) Find the Fourier sine transform of the function $f(x) = e^{-x}$.

6

Hence show that $\int_0^\infty \frac{x \sin mx}{1 + x^2} dx = \frac{\pi}{2} e^{-m}$

- B) Given $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$, using Fourier transform method evaluate 6

(i) $\int_0^\infty \frac{\sin \lambda \cos \lambda x}{\lambda} dx$ (ii) $\int_0^\infty \left(\frac{\sin \lambda}{\lambda} \right) d\lambda$

- C) Using Parseval's Identity Prove that 6

$$\int_0^\infty \frac{dt}{(a^2 + t^2)(b^2 + t^2)} = \frac{\pi}{2ab(a+b)}$$

Q5. Solve Any Two of the following.

- A) Solve $x(y-z)p + y(z-x)q = z(x-y)$ 6

- B) Use the method of separation of variables to solve the equation $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$. 6

- C) Solve $pz - qz = z^2 + (x+y)^2$ 6

Q6. Solve Any Two of the following.

- A) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. 6
Also find the corresponding analytic function.

- B) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is constant. 6

- C) Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$, where c is the circle $|z| = 3$ 6

*** End ***